INVESTIGATION OF HEAT TRANSFER IN A CROSS FLOW OF GAS OVER A RIGHT-ANGLED WEDGE

V. K. Fedorov and E. M. Litinskii

Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 2, pp. 171–176, 1965

Results are given of an analysis of experimental data on convective heat transfer based on the theory of local modeling. Laws of heat transfer are obtained for various angles of attack. A method of calculating the heat transfer is proposed for any law of variation of t_W.

Most of the work in the field of heat transfer on bodies of complex shape in forced convection has been devoted to determining the mean value of the heat transfer coefficient. At the same time, a knowledge of the local distribution of the heat transfer coefficient is required for correct solution of a certain number of scientific and engineering problems. This matter has been much less thoroughly investigated. In particular, very little attention has been devoted to the distribution of local heat transfer over a wedge in a cross flow. The well-known solution of Eckert [1] does not fill this gap, since it is valid only for a 45° angle of attack. Here [1], as elsewhere, the angle of attack is understood to be the acute angle between the velocity vector of the free stream and the plane containing the investigated wedge surface

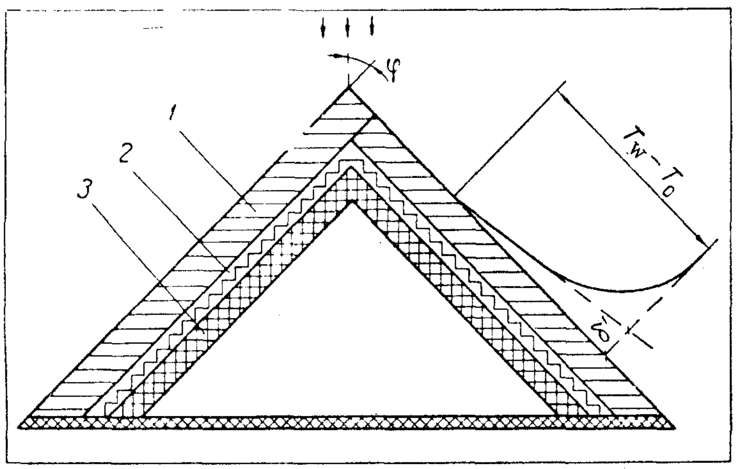

Fig. 1. Diagram of model: 1) copper plates; 2) heater; 3) textolite frame.

in the cross flow (Fig. 1). Moreover, this solution was derived for an infinite wedge, i. e., for a mathematical model very different from actual bodies. Finally, the resulting formula contains coefficients whose values are determined from experiment.

In the range of angles of attack from 90° to 45°, the experimental work of Drake [2] on heat transfer at a plate has a certain limited value. This author used Eckert's approach and therefore assumed a linear velocity distribution and hence equality of the heat transfer coefficient over the whole length of the plate. In reducing his experimental data, Drake used only constant values of α, relating to the central part of the model, on the erroneous assumption that the increased values of the heat transfer coefficient outside the central zone resulted from some kind of end effect.

As regards heat transfer at angles of attack below 45°, when a vortex region is created on the wedge surface investigated, little has been done, either experimental or theoretical.

The analysis and development of experimental material is much facilitated by using the theory of local modeling, the basic concepts of which were outlined in [3, 4].

The equation of a plane thermal boundary layer may be written in the following form:

$$\frac{d\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta \frac{1}{\bar{T}_W - 1} \frac{d(\bar{T}_W - 1)}{d\bar{x}} = \mathrm{St}\,\mathrm{Pe}_L. \tag{1}$$

To solve this equation, we need to know the heat transfer law relating the parameters St and Pe_θ.

To determine this law we may, firstly, use the equation of heat transfer for a flat isothermal plate [5].

Additionally, we may use the experimental data to compute local values of St and Pe_θ according to the formulas [6]

$$\mathrm{Pe}_\theta = \int_0^x q_W\, dx/(T_W - T_0)\,\lambda_0, \tag{2}$$

$$\mathrm{St} = \alpha/\rho_0 u_0 c_p g. \tag{3}$$

The first of the methods we have mentioned for establishing the law of heat transfer does not take into account the influence of various perturbing factors, and its use may lead to errors. Therefore, the experimental data that we obtained for angles of attack $90° \geqslant \varphi > 45°$ were processed according to (2) and (3), the velocity at the edge of the boundary layer, required for determination of local values of the Stanton number, being found from the static pressure distribution.

The results of our reduction of the experimental data are presented in Fig. 2, where the ordinate is the logarithm of the product $\mathrm{St}\,\mathrm{Pr}^{1/3}$. The latter factor was introduced to allow for the influence of the physical parameters of the medium on heat transfer.

Analysis of the graph obtained shows that, for angles of attack $90° \geqslant \varphi > 45°$ the heat transfer law may be expressed by the formula

$$\mathrm{St} = 0.36/\mathrm{Pe}_\theta\, \mathrm{Pr}^{1/3}. \tag{4}$$

Below, we examine the motion of a gas with uniform distribution of temperature and velocity in the free stream. The gas flow in the free stream will be considered isentropic with a geometric effect determined by the wedge geometry.

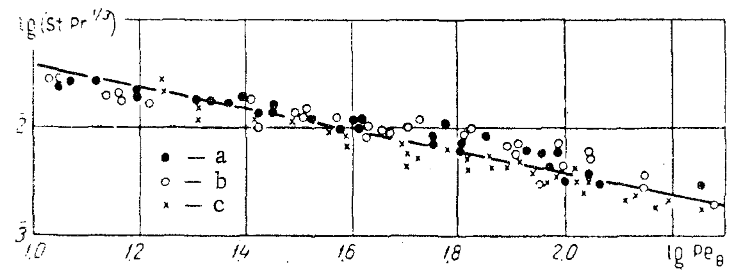

Fig. 2. The relation $\mathrm{St} = f(\mathrm{Pe}_\theta)$ for heat transfer in a cross flow over a right-angled wedge with: a) $\varphi = 90°$; b) 67.5°; c) 45°.

The velocity distribution at the edge of the boundary layer for the solution of (1) will be taken as follows: for angles of attack $90° \geqslant \varphi > 45°$ as for a plate immersed in a perfect fluid [7], and for the angle of attack $\varphi = 45°$ — directly from experiment.

Thus, for

$$u/u_0 = -\cos\varphi + \sin\varphi\, \bar{x}_1/\sqrt{1-\bar{x}_1^2}, \tag{5}$$

where $\bar{x}_1$ is the dimensionless distance from the middle of the wedge surface to the given point.

For an angle of attack $\varphi = 45°$, the velocity distribution at the edge of the boundary layer is described by the following equation [8]:

$$u/u_0 = C\bar{x}^{1/3}. \tag{6}$$

On the basis of the experimental data presented in Fig. 3, the value of coefficient C was taken as 1.2.

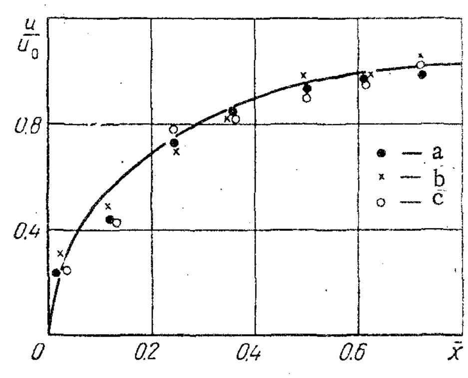

Fig. 3. Velocity distribution at the edge of the boundary layer for an angle attack $\varphi = 45°$ and a) $u_0 = 4.2$ m/sec; b) 6.5; c) 9.8.

To derive calculation formulas, we used in addition to (1), the heat transfer law (4) and the velocity distribution laws at the edge of the boundary layer (5) and (6).

The result for angles of attack $90° \geqslant \varphi > 45°$ is

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta \frac{d}{d\bar{x}} \ln(\bar{T}_W - 1) = 0.36\, \frac{\mathrm{Pe}_l}{\mathrm{Pe}_\theta\, \mathrm{Pr}^{1/3}} \left(-\cos\varphi + \frac{\sin\varphi\, \bar{x}_1}{\sqrt{1-\bar{x}_1^2}}\right), \tag{7}$$

and for the angle of attack $\varphi = 45°$

$$\frac{d\,\mathrm{Pe}_0}{d\bar{x}} + \mathrm{Pe}_0 \frac{d}{d\bar{x}} \ln(\bar{T}_W - 1) = \frac{0.36}{\mathrm{Pe}_0 \mathrm{Pr}^{1/3}} \mathrm{Pe}_L \cdot 1.2 \sqrt[3]{\bar{x}}. \tag{8}$$

In the above formulas and below, the subscripts l and L denote that the characteristic dimension has been taken as the half-length or the full length of the wedge surface, respectively.

The boundary conditions are

$$\text{at } \bar{x} = 0,\ \mathrm{Pe}_0 = 0,\ \bar{T}_W = \bar{T}_{W_1}.$$

After solving (7) and (8), we obtain the expressions

$$\mathrm{Pe}_0 = \frac{1}{\bar{T}_W - 1}\left(\frac{0.36\mathrm{Pe}_{l_0}}{\mathrm{Pr}^{1/3}}\right)^{1/2}\left(\int_0^{\bar{x}}\left(-\cos\varphi + \frac{\sin\varphi\,\bar{x}_1}{\sqrt{1-\bar{x}_1^2}}\right)(\bar{T}_W - 1)^2 d\bar{x}\right)^{1/2}, \tag{9}$$

$$\mathrm{Pe}_0 = \frac{1}{\bar{T}_W - 1}\left(\frac{0.36\mathrm{Pe}_{L_0}}{\mathrm{Pr}^{1/3}}\right)^{1/2}\left(\int_0^{\bar{x}} 1.2\sqrt[3]{\bar{x}}\ (\bar{T}_W - 1)^2 d\bar{x}\right)^{1/2}. \tag{10}$$

Using the heat transfer law (4) and formulas (9) and (10), we obtain equations for calculating the local heat transfer coefficients for a wedge in a cross flow of gas:

for angles of attack $90° \geq \varphi > 45°$

$$\mathrm{Nu}_l = \frac{0.43(\bar{T}_W - 1)\sqrt{\mathrm{Pe}_{l_0}}\left(-\cos\varphi + \sin\varphi\,\bar{x}_1/\sqrt{1-\bar{x}_1^2}\right)}{\left[\int_0^{\bar{x}}\left(-\cos\varphi + \sin\varphi\,\bar{x}_1/\sqrt{1-\bar{x}_1^2}\right)(\bar{T}_W - 1)^2 d\bar{x}\right]^{1/2} \mathrm{Pr}^{1/6}}, \tag{11}$$

for the angle of attack $\varphi = 45°$

$$\mathrm{Nu}_L = 0.43(\bar{T}_W - 1)\sqrt{\mathrm{Pe}_{L_0}}\ 1.2\sqrt[3]{\bar{x}}\left[\left(\int_0^{\bar{x}} 1.2\sqrt[3]{\bar{x}}\ (\bar{T}_W - 1)^2 d\bar{x}\right)^{1/2} \mathrm{Pr}^{1/6}\right]^{-1}. \tag{12}$$

Here and below the subscripts L_0 and l_0 denote criteria based on the free-stream parameters.

For the case when T_W = const, the calculation formulas take the form

for angles of attack $90° \geq \varphi > 45°$

$$\frac{\mathrm{Nu}_l}{\sqrt{\mathrm{Pe}_{l_0}}}\mathrm{Pr}^{1/6} = 0.43\left(-\cos\varphi + \frac{\sin\varphi(\cos\varphi + \bar{x})}{\sqrt{1-(\cos\varphi + \bar{x})^2}}\right)\Big\{-\cos\varphi\bar{x} + \sin\varphi\cos\varphi[\arcsin(\cos\varphi + \bar{x}) - \arcsin\cos\varphi] + \sin\varphi\Big[-\cos\varphi[\arcsin(\bar{x} + \cos\varphi) - \arcsin\cos\varphi] - \{[1-(\cos\varphi + \bar{x})^2]^{1/2} - (1-\cos^2\varphi)\}\Big]\Big\}^{-1/2}, \tag{13}$$

and for the angle of attack $\varphi = 45°$

$$\mathrm{Nu}_L = 0.55 \sqrt{\mathrm{Pe}_{L_0}} / \bar{x}^{1/3} \mathrm{Pr}^{1/6}. \tag{14}$$

The value of 1/2 for the exponent of the Peclet number in formulas (13) and (14), and also the nature of the temperature profile are evidence that in all these cases the boundary layer is laminar.

In Figs. 4a and 4b the results of calculations according to (13) and (14) are compared with experimental data. It can be seen from the graphs that there is quite satisfactory agreement between theory and experiment.

We also obtained experimental data on heat transfer at the angle of attack $\varphi = 0, 0°$, when vortex motion existed on the investigated surface of the wedge. Since the velocity at the edge of the boundary layer is, in this case, an unstable quantity, the experimental data could not be processed in St-Pe_θ coordinates.

Figure 4c shows the distribution of the experimental points for the case $\varphi = 0°$ in the parameters $\mathrm{Nu_L}$ and $\mathrm{Pe_L}$, based on the characteristic linear dimension of the model. The exponent 0.8 of the Peclet number is evidence that in this case the boundary layer is turbulent. The experimental points of Fig. 4c are approximated by the formula

$$\mathrm{Nu}_L = 0.031\, \mathrm{Pe}_L^{0.8} (1.4\bar{x} + 1). \tag{15}$$

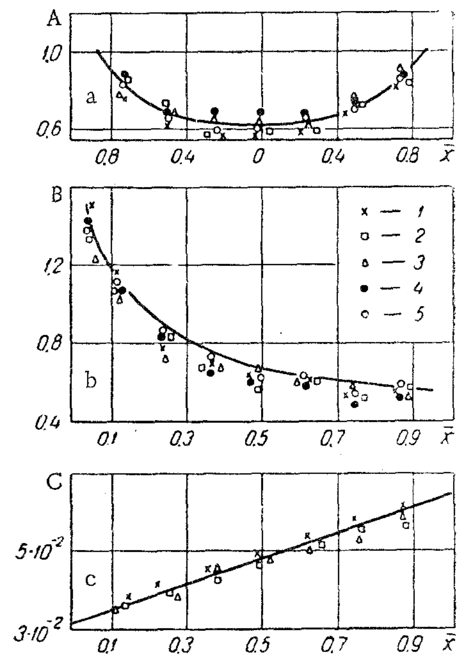

Fig. 4. Comparison of experimental data and the results of calculation a) according to (13); and b) according to (14); and c) the relation $\mathrm{Nu_L} = f(\mathrm{Pe_L})$ for heat transfer in a cross flow over a right-angled wedge at the angle of attack $\varphi = 0°$ – $A \equiv \mathrm{Nu}_l \mathrm{Pr}^{1/6} \mathrm{Pe}_{l_0}$; $B \equiv \mathrm{Nu_L} \mathrm{Pr}^{1/6} \mathrm{Pe}_{L_0}$; $C \equiv \equiv \mathrm{Nu_L}/\mathrm{Pe}_L^{0.8}$): 1–$u_0$ = 2.5 m/sec; 2–4.6; 3–6.0; 4–10.6; 5–13.4.

The test data were obtained on an experimental apparatus, the main part of which consisted of an open-circuit wind tunnel with a 600 × 600 mm working section. Two Prandtl tubes, connected to a micromanometer, and thermoelectric anemometers mounted in two mutually perpendicular directions in the tunnel entrance section were used to measure the velocity field in the undisturbed flow. The flow velocity was controlled using a special gate valve in the tunnel exhaust channel.

The model was mounted in the tunnel working section on rotating supports, which allowed the angle of attack to be varied and the model to be locked in any fixed position. The model used was a right-angled wedge consisting of two copper plates 10 mm thick mounted on a textolite frame. The model length was made equal to the tunnel height in order to avoid the formation of conical flow and to treat the problem as plane. The copper plates were heated by an electrical heater whose power was controlled by an autotransformer in a stabilized-voltage circuit. The copper plates were carefully polished in order to reduce the effect of roughness to a minimum. Copper-constantan thermocouples, diameter 0.15 mm, were located in the copper plates. The emf in all the thermocouples was determined by means of a potentiometer.

A movable microthermocouple, diameter 0.05 mm, was used to determine the temperature profile of the thermal boundary layer. Measurements were taken at five sections along the wedge surface. Temperature curves were constructed from the values obtained. The value of the heat transfer coefficient was determined from the formula $\alpha = \lambda/\delta'$. The determination of δ' is shown in Fig. 1. In view of the straight section on the temperature curve, its tangent could be drawn with sufficient accuracy. The heat flux density was determined by multiplying the local values of the heat transfer coefficient by the temperature difference between the free stream and the wall at the given section.

During the test, the velocity was varied in the range 0–14 m/sec, the wall temperature in the range 65–90° C, and the temperature of the medium in the range 18–21° C.

NOTATION

Pe_θ—Peclet number based on the characteristic thickness of the boundary layer and the velocity at the edge of the boundary layer; $\mathrm{Pe_L}$—Peclet number based on the characteristic dimension and the velocity at the edge of the boundary layer; $\bar{T}_W$—dimensionless wall temperature; $\bar{x}$—dimensionless coordinate; T_0—free-stream temperature; λ_0—thermal conductivity of the medium at the free-stream temperature; u_0—velocity at the edge of the boundary layer; α—local heat transfer coefficient; ρ_0—density of the medium at the free-stream temperature; λ—thermal conductivity of medium at the wall temperature; δ'—nominal thickness of boundary layer.

REFERENCES

1. E. Eckert, VDI—Forschungsheft, 416, 1942.
2. R. M. Drake, Journal of Applied Mechanics, 16, 1949.
3. V. M. Ievlev, DAN SSSR, vol. 36, no. 6, 1952.
4. V. M. Ievlev, DAN SSSR, vol. 36, no. 1, 1952.
5. V. K. Fedorov, IFZh [Journal of Engineering Physics], no. 2, 1965.
6. S. I. Kosterin, A. I. Leont'ev, and V. K. Fedorov, Teploenergetika, no. 5, 1962.
7. N. E. Kochin, I. A. Kibel, and N. V. Rose, Theoretical Hydromechanics [Russian translation], Gostekhizdat, 1948.
8. H. Schlichting, Boundary Layer Theory [Russian translation], IL, 1956.

18 January 1965

Institute of Building Physics, Moscow